# Kinetic Isotope Effects and Aliphatic Diazo-compounds. Part III. $\dagger$ Solvent Isotope Effects for 3-Diazobutan-2-one and Ethyl Diazopropionate 

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#### Abstract

Data are reported for the isotopic analysis of the products of the acid-catalysed decomposition of 3-diazobutan-2one and ethyl diazopropionate and for the rate of reaction for both compounds in mixtures of $\mathrm{H}_{2} \mathrm{O}$ and $\mathrm{D}_{2} \mathrm{O}$. The kinetic data are analysed to obtain the fractionation factors for the proton transferred to carbon and for the two secondary solvent protons. The fractionation factors from the product analysis agree with the factors for the proton transferred to carbon. A simple analytical correction for the breakdown of the rule of the geometric mean which will hold for most $A-S_{\mathrm{E}} 2$ transition states is presented and applied to the data.

The data for 3-diazobutan-2-one were obtained by the differential method and their higher precision allows the investigation of models other than the simple one for the $A-S_{\mathrm{E}} 2$ transition state and in particular the inclusion of transition-state solvation terms. The results are not completely conclusive but suggest that the simple model is the most probable one. The values of $\alpha$ deduced from the secondary solvent isotope effect do not agree with those from the Bronsted catalysis law.


The study of the variation of reaction rate with $x$, the atom fraction of deuterium, in mixtures of $\mathrm{H}_{2} \mathrm{O}$ and $\mathrm{D}_{2} \mathrm{O}$ can give detailed information about the nature of the transition state. The variation is best described by fractionation factor theory. ${ }^{1,2}$ For an $A-S_{\mathrm{E}} 2$ reaction the simplest formulation, which we shall call the simple


Scheme 1
model, of the reaction is shown in Scheme 1, which leads to equation (1). The value of $l$ has been measured

$$
\begin{equation*}
\frac{k_{x}}{k_{0}}=\frac{\left(1-x+\phi_{1} x\right)\left(1-x+\phi_{2} x\right)^{2}}{(1-x+l x)^{3}} \tag{1}
\end{equation*}
$$

to be $0 \cdot 69 .{ }^{3-5}$ Analysis of the variation of $\left(k_{x} / k_{0}\right)$ with $x$ then allows in theory the determination of $\phi_{1}$ and $\phi_{2}$. In practice one requires very accurate kinetic data to obtain an unambiguous determination of $\phi_{1}$ and $\phi_{2}$ from the kinetics alone. In many reactions the L-S bond once formed in the transition state does not further exchange and so $\phi_{1}$ can often be measured from the isotopic content ${ }^{2}$ of the product measured as $\phi_{\mathrm{P}}$ [equation (2)]. A recent paper ${ }^{6}$ reviews values

$$
\begin{equation*}
\phi_{\mathrm{P}}=\frac{[\mathrm{DS}]}{[\mathrm{HS}]}\left(\frac{1-x}{x}\right) \tag{2}
\end{equation*}
$$

of $\phi_{1}$ and $\phi_{2}$ measured by these methods. For the
$\dagger$ Part II, W. J. Albery, J. S. Curran, and A. N. CampbellCrawford, preceding paper.
${ }^{1}$ V. Gold, Trans. Faraday Soc., 1960, 56, 255.
2 A. J. Kresge, Pure Appl. Chem., 1964, 8, 243.
3 V. Gold, Proc. Chem. Soc., 1963, 141.
${ }^{4}$ A. L. Allred and A. J. Kresge, J. Amer. Chem. Soc., 1963, 85, 1541 .
${ }_{5}$ K. Heinzinger and R. E. Weston, J. Phys. Chem., 1964, 68, 744.
acid-catalysed decomposition of $\mathrm{N}_{2} \cdot \mathrm{CMe} \cdot \mathrm{COMe}$ and $\mathrm{N}_{2} \cdot \mathrm{CMe} \cdot \mathrm{CO}_{2} \mathrm{Et}$ we have succeeded in measuring the kinetic ratios accurately enough to determine $\phi_{1}$ and $\phi_{2}$ from equation (1); we have also measured $\phi_{\mathrm{P}}$ from equation (2). The differential method ${ }^{7,8}$ was used for the diazo-ketone but not for the diazo-ester where the reaction rates were too fast.

## EXPERIMENTAL

Materials and Solutions.-Most chemicals have been described. ${ }^{9} \quad \mathrm{D}_{2} \mathrm{O}$ was supplied by Norsk Hydro Electrisk and was stated to contain 99.7 g of $\mathrm{D}_{2} \mathrm{O}$ per 100 g of liquid. The H content of the $\mathrm{D}_{2} \mathrm{O}$ was measured by examining the ${ }^{1} \mathrm{H}$ n.m.r. spectrum of a sample of the $\mathrm{D}_{2} \mathrm{O}$ with a small quantity of MeOH (ca. 0.03 g in 5 g of $\mathrm{D}_{2} \mathrm{O}$ ) as an internal standard. Typical values of 99.65 g and 99.66 g of $\mathrm{D}_{2} \mathrm{O}$ per 100 g were obtained which agreed well with the manufacturers' figure. Acid solutions for kinetic experiments were made up by weight from a stock solution of $0 \cdot 10 \mathrm{~m}-$ $\mathrm{HClO}_{4}$ and $\mathrm{H}_{2} \mathrm{O}$ or $\mathrm{D}_{2} \mathrm{O}$. For ethyl diazopropionate $\left[\mathrm{L}_{3} \mathrm{O}^{+}\right]$was throughout 1.00 mm . For 3-diazobutan-2-one, to obtain the full benefit of the differential method two stock acid solutions were made up in $\mathrm{H}_{2} \mathrm{O}$ and $\mathrm{D}_{2} \mathrm{O}$ with $\left[\mathrm{H}_{3} \mathrm{O}^{+}\right]=2 \mathrm{~mm}$ and with $\left[\mathrm{D}_{3} \mathrm{O}^{+}\right] \simeq 5 \mathrm{~mm} \simeq 2 \cdot 5\left[\mathrm{H}_{3} \mathrm{O}^{+}\right]$; reaction solutions were mixtures of these $\mathrm{H}_{3} \mathrm{O}^{+}$and $\mathrm{D}_{3} \mathrm{O}^{+}$ solutions. The factor of 2.5 offset the isotope effect so that the actual rate of reaction in the $\mathrm{H}_{2} \mathrm{O}-\mathrm{D}_{2} \mathrm{O}$ mixture was always close to the rate in the pure $\mathrm{H}_{2} \mathrm{O}$ with which it was being compared.

Kinetic Measurements.-The apparatus and technique, both single ${ }^{9}$ and differential, ${ }^{8}$ were as described. In the differential runs twice as much of the diazo-ketone was added to the pure $\mathrm{H}_{2} \mathrm{O}$ solution.

Isotopic Analysis of Products.-The isotopic analysis was carried out by ${ }^{1} \mathrm{H}$ n.m.r. on the major product, LO.CL$\mathrm{Me} \cdot \mathrm{COR}$ where R is Me or OEt. 10 ml of $\mathrm{H}_{2} \mathrm{O}-\mathrm{D}_{2} \mathrm{O}$ mixture containing ca. $5 \mathrm{~mm}-\mathrm{L}_{3} \mathrm{O}^{+}$were made up by weight.
${ }^{6}$ R. A. More O'Ferrall, G. W. Koeppl, and A. J. Kresge, J. Amer. Chem. Soc., 1971, 93, 9.

7 W. J. Albery and B. H. Robinson, Trans. Faraday Soc., 1969, 65, 980.
${ }^{8}$ 'W. J. Albery and M. H. Davies, Trans. Faraday Soc., 1969, 65, 1066 .
${ }_{9}$ W. J. Albery, A. N. Campbell-Crawford, and K. S. Hobbs, J.C.S. Perkin II, 1972, 2180.

Diazo-compound ( 1.5 ml ) was added gradually dropwise during $2-3 \mathrm{~h}$; the slow addition kept the concentration of diazo-compound low throughout the decomposition. Evolution of $\mathrm{N}_{2}$ ceased after 4-6 h. In the case of 3-diazobutan-2-one, the acetoin was extracted into ether by continuous liquid extraction. After being dried $\left(\mathrm{Na}_{2} \mathrm{SO}_{4}\right)$ and the ether having being distilled off on a water-bath, the acetoin [HOCLMeCOMe] was distilled over in a micro-distillation apparatus at $30{ }^{\circ} \mathrm{C}$ under a pressure of 14 mmHg . The ${ }^{1} \mathrm{H}$ n.m.r. spectra were obtained on a Varian A60A instrument at $10^{\circ} \mathrm{C}$. It was necessary to use the lower temperature to separate the CH quartet from the OH resonance. The unsplit $\mathrm{CH}_{3}$ of the $\mathrm{CH}_{3} \mathrm{CO}$ was used as the internal standard. It was necessary to check that enolisation did not exchange either the CL proton or the $\mathrm{CH}_{3}$ of the internal standard. Over a period of weeks no exchange could be detected in protioacetoin dissolved in $10^{-2} \mathrm{M}_{-} \mathrm{D}_{3} \mathrm{O}^{+}$in $\mathrm{D}_{2} \mathrm{O}$. In $10^{-1} \mathrm{M}-$ $\mathrm{D}_{3} \mathrm{O}^{+}$a slight drop was found in the $\mathrm{CH}_{3}$ signal. This agrees with a half-time ${ }^{10}$ of 1.5 days for the $\mathrm{CH}_{3}$ and 5 days for the CL site in $2 \mathrm{M}-\mathrm{D}_{2} \mathrm{SO}_{4}$. These rates are far too slow to affect our experiments. Concerning ethyl diazopropionate, in the ${ }^{1} \mathrm{H}$ n.m.r. spectrum of ethyl lactate, $\mathrm{LO} \cdot \mathrm{CLMe} \cdot \mathrm{CO}_{2} \mathrm{Et}$, the $\mathrm{CH}_{2}$ signal is inextricably entwined with that from the CL. The product was therefore acetylated by use of the same procedure as previously ${ }^{8}$ except that acetyl chloride was used as the acetylating agent. The CL and the $\mathrm{CH}_{2}$ signals are then well separated. The ${ }^{1} \mathrm{H}$ n.m.r. spectra were measured on a PerkinElmer R12 instrument. To check that no exchange took place, first all-protioethyl lactate was stirred for several hours in $10^{-3} \mathrm{M}-\mathrm{D}_{3} \mathrm{O}^{+}$and then acetylated; the ratio of the integrated $\mathrm{CH}_{2}$ and CL peaks was 2:1. Secondly ethyl diazopropionate was decomposed in $\mathrm{D}_{2} \mathrm{O}$; the resulting ethyl acetyl-lactate had less than $\frac{1}{2} \%$ of CH present. In measuring the ${ }^{1} \mathrm{H}$ n.m.r. spectra care was taken to avoid integrating spinning side bands and ${ }^{13} \mathrm{C}$ satellites. Multiple (at least 14) upfield and downfield sweeps were carried out.

Solubility Measurements.-The solubility of the diazocompounds in $\mathrm{H}_{2} \mathrm{O}-\mathrm{D}_{2} \mathrm{O}$ mixtures was measured by partition between 25 ml of a 1 mm solution of the diazocompound in $\mathrm{CCl}_{4}$ and 5 ml of $\mathrm{L}_{2} \mathrm{O}$ at pH 10 (to prevent decomposition). After equilibration at $25{ }^{\circ} \mathrm{C}$ the optical density of the aqueous phase was measured. Since less than $2 \%$ of the diazo-compound dissolved in the aqueous phase and since there was very little change in the solubility the chemical potential of the diazo-compound in the organic phase is the same in each experiment. Hence the ratio of the solubilities is given by the ratio of optical densities.

## RESULTS

Differential Runs for 3-Diazobutan-2-one.-In order to test the differential method and in particular that there was no systematic temperature difference between the two cell compartments, runs were carried out in the same $\mathrm{H}_{3} \mathrm{O}^{+}$ solution; the only difference between the resulting solutions was that one cell had twice as much diazo-ketone (two drops of 3 -diazobutan-2-one solution rather than one). This meant that the acid concentration was $c a \frac{1}{2} \%$ more dilute in the cell with more ketone. Typical results are
${ }^{10}$ J. W. Marsinon, J. Lubach, and W. Drenth, Rec. Trav. chim., 1969, 88, 193.
plotted in Figure (1), according to equation ${ }^{8}$ (3), where $k^{\prime}=k(1+\delta)$, and $D_{t}^{\prime \prime}=D_{t}^{\prime}-D_{t}$, and $D_{t}^{\prime}$ and $D_{t}$ are
$y=\log \left[\frac{D^{\prime \prime}{ }_{t}-D^{\prime \prime}{ }_{\infty}}{D_{t}+\Delta t-D_{\infty}}+\exp (k \Delta t)\right]=$ constant $-\frac{\delta k t}{2 \cdot 303}$
the optical densities at time $t$ measured against a cell containing solvent and $D_{t}^{\prime}>D_{t} ; k^{\prime}$ and $k$ are the respective first-order rate constants and $\Delta t$ is a small constant time interval between the measurement of $D^{\prime \prime}$ in the


Figure 1 Typical results by differential method for 3-diazo-butan-2-one plotted by equation (3); (a) blank run with difference $c a .0 .5 \%$; (b), $x=0.196$ with difference $c a .13 \%$
differential mode and $D_{t+\Delta t}$ in the normal single mode. $D_{t}$ Was the $\mathrm{L}_{2} \mathrm{O}$ solvent and $D_{t}^{\prime}$ the pure $\mathrm{H}_{2} \mathrm{O}$. During the run readings are taken alternately of $D^{\prime \prime}{ }_{t}$ and $D_{t}+\Delta t$, and these are then paired together. The value of $k$ required to calculate $y$ and $\delta$ from $\delta k$ is found from a normal first-order plot of $\log \left(D_{t}-D_{\infty}\right)$ against time. The reason $D_{0}^{\prime}$ is made twice $D_{0}$ is so that throughout the run $D^{\prime \prime} \sim D$; this leads to cancellation in slide-wire errors. The gradient of equation (3) is a direct measure of $\delta k$ and hence of the small difference between the rate constants. In a blank run $\delta=c a$. $0 \cdot 005$. Figure 1 also shows another typical run where $\delta=0.132$ in an $\mathrm{H}_{2} \mathrm{O}-\mathrm{D}_{2} \mathrm{O}$ mixture of $x=0 \cdot 191$. To calculate $k_{x} / k_{0}$ we write equation (4). The results are in Table 1.

$$
\begin{equation*}
\frac{k_{x}}{k_{0}}=\frac{\left[\mathrm{H}_{3} \mathrm{O}^{+}\right]}{\left[\mathrm{L}_{3} \mathrm{O}^{+}\right](1+\delta)} \tag{4}
\end{equation*}
$$

Kinetic Results for Ethyl Diazopropionate.--The results for $k_{x} / k_{0}$ are given in Table 2. Each result was the mean of three determinations. The value of $k_{0}$ is calculated from all the kinetic data as discussed below.

Isotopic Analysis of the Products.-Results for $\phi_{\mathrm{P}}$ for both compounds are given in Table 3. The values of $\phi_{P^{*}}{ }^{*}$ have been corrected through equation (5) for the breakdown of the rule of the geometric mean ${ }^{11}$ in the solvent and $\phi_{\mathrm{P}}$ refers to a standard state of $x=0.50$. Since

$$
\begin{equation*}
\phi_{\mathrm{P}}=\phi_{\mathrm{P}}{ }^{*}\left[1.00-0.06\left(\frac{1}{2}-x\right)\right] \tag{5}
\end{equation*}
$$

$\phi_{\mathrm{P}}=c a .0 .20$ the experiments were carried out for $x>0.50$ so that the $\mathrm{D}: \mathrm{H}$ ratio in the product was not too small.

Table 1
Differential runs for 3-diazobutan-2-one

| $\stackrel{x}{0.000}$ | $\left[\mathrm{H}_{3} \mathrm{O}^{+}\right] /\left[\mathrm{L}_{3} \mathrm{O}^{+}\right]$ | $10^{2} \delta$ -0.7 | $k_{x} / k_{0} \quad$ Mean | $k_{x} / k_{0}(\mathrm{Calc}$. 0.000 |
| :---: | :---: | :---: | :---: | :---: |
| 0.000 | 0.993 | $-0.7 \pm 0.2$ | 1.000 |  |
| Blank | $0 \cdot 992$ | $-0.8 \pm 0 \cdot 1$ | 1.000 |  |
| runs | $0 \cdot 995$ | $-0.1 \pm 0 \cdot 1$ | $0.996 \quad 0.999$ | 1.00 |
|  | 0.995 | $-0.9 \pm 0.2$ | $1.004 \pm 0.001$ |  |
|  | $0 \cdot 994$ | $-0.3 \pm 0.2$ | 0.997 |  |
|  | 0.995 | $-0.7 \pm 0.1$ | 1.002 |  |
| $0 \cdot 191$ | 0.810 | $-13.2 \pm 0 \cdot 1$ | 0.933 |  |
|  | 0.810 | $-13.5 \pm 0 \cdot 2$ | $0.937 \quad 0.935$ | 0.934 |
|  | 0.810 | $-13.4 \pm 0 \cdot 2$ | $0.935 \pm 0.001$ |  |
| $0 \cdot 394$ | 0.677 | $-19 \cdot 2 \pm 0 \cdot 2$ | 0.839 |  |
|  | $0 \cdot 677$ | $-19 \cdot 3 \pm 0 \cdot 1$ | $0.840 \quad 0.839$ | $0 \cdot 846$ |
| $0 \cdot 607$ | $0 \cdot 577$ | $-21 \cdot 8 \pm 0.2$ | 0.738 |  |
|  | $0 \cdot 577$ | $-21.0 \pm 0.3$ | $0.730 \quad 0.735$ | 0.725 |
|  | $0 \cdot 577$ | $-21.9 \pm 0 \cdot 2$ | $0.738 \pm 0.002$ |  |
| 0.798 | 0.510 | $-13.9 \pm 0.3$ | 0.592 |  |
|  | $0 \cdot 510$ | $-13 \cdot 8 \pm 0.2$ | $0.592 \quad 0.592$ | $0 \cdot 584$ |
| 0.992 | $0 \cdot 456$ | $+15 \cdot 7 \pm 0 \cdot 2$ | 0.394 |  |
|  | $0 \cdot 456$ | $+15 \cdot 8 \pm 0.1$ | $0.394 \quad 0.394$ | $0 \cdot 394$ |
|  | $0 \cdot 456$ | $+15 \cdot 8 \pm 0 \cdot 2$ | 0.394 |  |

Table 2
Solvent isotope effect for ethyl diazopropionate $k_{0}=19 \cdot 0 \pm 0 \cdot 11 \mathrm{~mol}^{-1} \mathrm{~s}^{-1}$

$k_{x} / k_{0}(\mathrm{obs})$
$0.999 \pm 0.006$
$0.949 \pm 0.009$
$0.948 \pm 0.015$
$0.895 \pm 0.003$
$0.792 \pm 0.010$
$0.660 \pm 0.002$
$0.508 \pm 0.007$
$0.406 \pm 0.003$
$k_{x} / k_{0}($ calc.
1.000
0.949
0.943
0.890
0.785
0.670
0.498
0.401

Table 3
Product fractionation factors

| 3-Diazobutan-2-one |  |
| :---: | :---: |
| CD/CH | $\phi_{\mathbf{P}}$ |
| $0 \cdot 241$ | $0 \cdot 220$ |
| $0 \cdot 255$ | $0 \cdot 230$ |
| 0.310 | $0 \cdot 180$ |
| $0 \cdot 606$ | $0 \cdot 205$ |
| $0 \cdot 814$ | $0 \cdot 205$ |
| Mean | $0.208 \pm 0.008$ |
| Ethyl diazopropionate |  |
| $0 \cdot 200$ | $0 \cdot 190$ |
| 0.479 | $0 \cdot 222$ |
| 0.785 | $0 \cdot 200$ |
| Mean | $0.204 \pm 0.009$ |

Table 4
Solubility ratios
3-Diazobutan-2-one
Solubility in $\mathrm{L}_{2} \mathrm{O}$
Solubility in $\mathrm{H}_{2} \mathrm{O}$
$1.00_{0}$
0.99 $0.99_{3}$
0.98
$0.98_{3}$
$0.97_{3}$
Ethyl diazopropionate

Solubility Measurements.-The ratios of the solubility of 3 -diazobutan-2-one and ethyl diazopropionate in $\mathrm{L}_{2} \mathrm{O}$ compared to $\mathrm{H}_{2} \mathrm{O}$ are given in Table 4.

## DISCUSSION

We first analyse the data in Tables 1 and 2 according to equation (1). The solubility experiments suggest that we can ignore any reactant fractionation factors describing the solvation of the diazo-compound; to a first approximation they may cancel with those in the


Figure 2 Plot of $y / x$ against $x$ from equation (6) for 3-diazobutan-2-one
transition state. Equation (l) is rearranged to give (6) where $\theta_{n}=1-\phi_{n}$. On the left-hand side the

$$
\begin{align*}
& y=1-k_{x} / k_{0}(1-x+l x)^{3}-\theta_{1} \theta_{2}^{2} x^{3}= \\
&  \tag{6}\\
& \left(\theta_{1}+2 \theta_{2}\right) x-\theta_{2}\left(2 \theta_{1}+\theta_{2}\right) x^{2}
\end{align*}
$$

$\theta_{1} \theta_{2}{ }^{2} x^{3}$ term is small since even at $x=1, \theta_{1} \theta_{2}{ }^{2} /$ R.H.S. $=$ ca. 0.04 for $\theta_{1}=0.80$ and $\theta_{2}=0.20$. Thus this term is allowed for by successive iteration in the values of $y$ until the values of $\theta_{1}$ and $\theta_{2}$ converge to within $\frac{1}{2} \%$. If one divides equation (6) by $x$ one can obtain a linear plot ${ }^{11}$ of $y / x$ against $x$. Figure 2 shows the data for 3-diazobutan-2-one plotted in this way. However, we now think that this procedure gives too much weight to the rather uncertain points when both the left-hand side of equation (6) and $x$ tend to zero. We have therefore found values of $a$ and $b$ which minimise the square of the deviations in equation (7). We

$$
\begin{equation*}
y=a x+b x^{2} \tag{7}
\end{equation*}
$$

then solve the quadratic equations for $\theta_{1}$ and $\theta_{2}$ taking

[^0]the roots for which $\theta_{1}>\theta_{2}$ as in equation (8) and (9).
\[

$$
\begin{align*}
& \theta_{1}=\frac{1}{8}\left[a+2\left(a^{2}+3 b\right)^{\frac{1}{2}}\right]  \tag{8}\\
& \theta_{2}=\frac{1}{3}\left[a-\left(a^{2}+3 b\right)^{\frac{1}{2}}\right] \tag{9}
\end{align*}
$$
\]

We have also worked out the standard errors on the values of $\theta$ according to equation (10), where $d_{i}$ is the

$$
\begin{equation*}
E_{\theta_{m}}=\sqrt{ }\left(\frac{\sum_{1}^{n} d_{i}^{2}}{(n-2)}\right) \sqrt{ }\left[\sum_{1}^{n}\left(\frac{\partial \theta_{m}}{\partial y_{i}}\right)^{2}\right] \tag{10}
\end{equation*}
$$

difference for the $i$ th run between $y_{\text {obs }}$ and $y_{\text {calc. }}$. The algebraic equations are given in Appendix 1.

For ethyl diazopropionate we have used a similar treatment; it is more complicated, since the raw data do not consist of rate ratios from the differential method, but they are single rate constants as a function of $x$. Most previous treatments of solvent isotope data take as knowns the values of $k$ at $x=0$ and in many cases the
down in the reactant $\mathrm{L}_{3} \mathrm{O}^{+}$. The factors for the transition state are shown in Scheme 2. $\mathrm{L}_{3} \mathrm{O}^{+}$is a special case of this Scheme with $p_{1}=p_{2}$ and $\phi_{1}=\phi_{2}=l$.

In our treatment we did not know the values of $p_{1}, p_{2}$, or $p_{3}$ although we suggested that $p_{1}$ would be small for symmetrical and product-like transition states and that $p_{2}$ would lie between $p_{3}$ and 0.03 . More O'Ferrall, Koeppl, and Kresge ${ }^{6}$ have recently attempted to calculate these quantities. Their scheme of fractionation factors is identical with ours but they express it in the different notation of equations (15) and (16).

$$
\begin{align*}
& p_{3}-p_{1}=\frac{1}{2}\left(1-R_{1}\right)  \tag{15}\\
& p_{3}-p_{2}=\frac{1}{4}\left(1-R_{1}\right)-\frac{1}{2}\left(1-R_{2}\right) \tag{16}
\end{align*}
$$

From their calculated values of $R_{1}$ and $R_{2}$ and of $p_{3}=0.03$ we can obtain values of our parameters and thus combine their calculations with our simple analytical expression.

In fact in equation (14), since $p_{1}$ is small for symmetrical

| HHXH | $\bar{\phi}_{2}\left(1+p_{2}\right)^{-1}$ | HDXH |
| :---: | :---: | :---: |
| $\phi_{1}\left(1+p_{1}\right)^{-1}$ |  | $\phi_{1}$ |
| HHXD | $\bar{\phi}_{2}\left(1+p_{3}\right)^{-1}\left(1+p_{1}\right)$ | HDXD |
|  |  | SCHEME 2 |

$$
\begin{array}{cc}
\bar{\phi}_{2}\left(1+p_{2}\right)\left(1+p_{1}\right)^{-1} & \text { DDXH } \\
\phi_{1}\left(1+p_{1}\right)
\end{array}
$$

value of $k$ at $x=1$ as well. Gold has discussed the validity of this procedure ${ }^{12}$ and we prefer to give equal weight to all our runs. Hence from equation (6) we write (11), and $y$ is fitted to equation (12) by iterating

$$
\begin{gather*}
y=k_{x}(1-x+l x)^{3}+k_{0} \theta_{1} \theta_{2}{ }^{2} x^{3}=k_{0}\left[1-\left(\theta_{1}+\right.\right. \\
\left.\left.2 \theta_{2}\right) x+\theta_{2}\left(2 \theta_{1}+\theta_{2}\right) x^{2}\right]  \tag{l1}\\
y=a^{\prime}+b^{\prime} x+c^{\prime} x^{2} \tag{12}
\end{gather*}
$$

for the small term in $y$ till $k_{0}, \theta_{1}$, and $\theta_{2}$ converge to within $\frac{1}{2} \%$. The equations are given in Appendix 2.* Values for the fractionation factors and their standard errors calculated from equation (6) and (11) are given in Table 6. These are the simple equations which are usually adequate to describe $A-S_{\mathrm{E}} 2$ reactions, but owing to the precision of our data we must discuss two further corrections. The first of these is the breakdown of the rule of the geometric mean. We have suggested a simple analytical correction [equation (13)] ${ }^{12}$ which is accurate to within two parts in a thousand. $g(x)$ Is given by equation (14), and $p_{1}$ and $p_{2}$ describe the

$$
\begin{align*}
\frac{k_{x}}{k_{0}} & =\frac{\left(1-x+\phi_{1} x\right)\left(1-x+\phi_{2} x\right)^{2}}{(1-x+l x)^{3}}[1+g(x)]  \tag{13}\\
g(x) & =x(1-x)\left[3 p_{3}-2 p_{2}-p_{1}\left\{\frac{\phi_{1} \phi_{2}{ }^{-1}(1-x)+\phi_{2} x}{\left(1-x+\phi_{2} x\right)}\right\}\right] \tag{14}
\end{align*}
$$

breakdown for the transition state and $p_{3}$ the break-

[^1]and product-like transition states $(x>0.3)$ we can write equation (17) with a maximum error of less than one part in a thousand:
\[

$$
\begin{equation*}
1+g(x)=1+P x(1-x) \tag{17}
\end{equation*}
$$

\]

where $P=\left(1-R_{1}\right)-\left(1-R_{2}\right)=3 p_{3}-2 p_{2}-p_{1} \bumpeq$ $p_{3}=0.03$. Table 5 shows values of $\left[\left(1-R_{1}\right)-\right.$ $\left.\left(1-R_{2}\right)\right]$ and for these transition states $(\alpha>0.3)$,

## Table 5

Correction function for breakdown of rule of geometric mean from data in ref. 6

| Bond order | 0 0.125 0.25 0.375 0.5 0.625 0.75 0.875 <br> $10^{2} P$        | 0 | 0.2 | 1.7 | 3.6 | 3.3 | 3.2 | 3.3 | 3.1 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 3.0 |  |  |  |  |  |  |  |  |  |

$P \bumpeq 0.03$. We can reach the same conclusion more directly by following our original arguments; we put $p_{3}=0.03, p_{2}$ between 0.03 and $p_{3}$, and $p_{1}=$ 0.0 . Therefore to allow for a correction for the breakdown of the rule of the geometric mean we multiply the observed $k_{x} / k_{0}$ or $k_{x}$ by $[1-0.03 x(1-x)]$. This correction should hold for most $A-S_{\mathrm{E}} 2$ transition states.

The second correction that we have to allow for in the particular case of the diazo-compounds is the interference from the second transition state. ${ }^{13}$ The full expression for the rate constant ${ }^{13}$ is (18) or (19) where $\phi_{s}$ describes the possible fractionation ${ }^{14}$ in the second

[^2]transition state. In fact it makes very little difference to the final results whether we put $\phi_{\mathrm{S}}=1.0$ or 0.88 . Again the correction is applied by multiplying $k_{x} / k_{0}$ or $k_{x}$ by the square bracket of equation (19).
\[

$$
\begin{align*}
& k_{\mathrm{obs}}=\frac{k_{\mathrm{L}^{+}}}{1+k_{\mathrm{L}_{\mathrm{t}} \mathrm{O}} / k_{2, \mathrm{~L}}}  \tag{18}\\
& k_{\mathrm{L}^{+}}=k_{\mathrm{obs}, x}\left[1+\frac{k_{\mathrm{H}, \mathrm{O}}^{\prime}}{k_{2, \mathrm{H}}} \frac{\left(1-x+\phi_{1} x\right)\left(1-x+\phi_{2} x\right)^{2}}{\left(1-x+\phi_{\mathrm{S}} x\right)^{2}}\right] \\
& \bumpeq k_{\mathrm{obs}, x}\left[1+\frac{k_{\mathrm{H}, \mathrm{O}}^{\prime}}{k_{2 . \mathrm{B}}} \frac{k_{\mathrm{obs}, x}}{k_{\mathrm{obs} .0}} \frac{(1-x+l x)^{3}}{\left(1-x+\phi_{S} x\right)^{2}}\right] \tag{19}
\end{align*}
$$
\]

The effects of these corrections and of performing the calculation with a different value of $l=0.70$ are shown in Table 6. Tables 1 and 2 compare observed and calculated data for the asterisked row in Table 6.

Table 6

| Fractionation factors |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{gathered} P \\ 0.00 \end{gathered}$ | 3-Diazobutan-2-one |  |  |  |  |  |
|  | $l$ | $k^{\prime} \mathrm{H}_{\mathbf{0}} / k_{\text {H }}$ | $10^{3} \phi_{1}$ | $10^{3} \phi_{2}$ | $\alpha_{L}{ }^{+}$ | $10^{3} \sigma$ |
|  | $0 \cdot 69$ | $0 \cdot 000$ | $210 \pm 3$ | $778 \pm 4$ | $0.32 \pm 0.014$ | 1.7 |
|  | $0 \cdot 69$ | 0.009 | $211 \pm 3$ | $774 \pm 4$ | $0.31 \pm 0.01{ }_{4}$ | 1.7 |
|  | 0.70 | 0.009 | $213 \pm 3$ | $789 \pm 4$ | $0.33 \pm 0.01{ }_{5}$ | 1.7 |
| 0.03 | 0.69 | 0.000 | $219 \pm 4$ | $765 \pm 5$ | $0.28 \pm 0.01_{8}$ | $2 \cdot 0$ |
|  | $0 \cdot 69$ | 0.009 | $219 \pm 4$ | $762 \pm 5$ | $0.27 \pm 0.01$ ¢ | 2.0* |
|  | 0.70 | 0.009 | $220 \pm 4$ | $777 \pm 5$ | $0.29 \pm 0.01$, | 2.0 |
|  | Product analysis $208 \pm 8$ |  |  |  |  |  |


| Ethyl diazopropionate |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.00 | 0.69 | 0.000 | $208 \pm 9$ | $785 \pm 12$ | $0.35 \pm 0.04$ | 2.6 |
|  | 0.69 | 0.010 | $209 \pm 9$ | $780 \pm 12$ | $0.33 \pm 0.04$ | 2.6 |
|  | 0.70 | 0.010 | $211 \pm 9$ | $795 \pm 13$ | $0.36 \pm 0.04$ | 2.6 |
| 0.03 | 0.69 | 0.000 | $216 \pm 9$ | $772 \pm 12$ | $0.30 \pm 0.04$ | 2.6 |
|  | 0.69 | 0.010 | $217 \pm 9$ | $768 \pm 12$ | $0.29 \pm 0.04$ | $2.6 *$ |
|  | 0.70 | 0.010 | $218 \pm 9$ | $782 \pm 12$ | $0.32 \pm 0.04$ | 2.6 |
|  |  |  |  |  |  |  |
| Product analysis $204 \pm 9$ |  |  |  |  |  |  |
|  | $*$ Used to calculate results in Tables 1 and 2. |  |  |  |  |  |

The values of $\alpha_{L^{+}}$in Table 6 are calculated from the relationship (20) suggested by Gold. ${ }^{1}$ In order to

$$
\begin{equation*}
\phi_{2}=l^{1-a_{L^{+}}} \tag{20}
\end{equation*}
$$

allow a comparison to be made with the other results in the compilation of More O'Ferrall et al. ${ }^{6}$ we have also calculated their parameter as given in equation (21).

$$
\begin{equation*}
\sigma=\left\{\sum_{1}^{n}\left[\left(k_{x} / k_{0}\right)_{\text {obs }}-\left(k_{x} / k_{0}\right)_{\text {calc }}\right]^{2} / n(n-1)\right\}^{\frac{1}{2}} \tag{21}
\end{equation*}
$$

The value of $\sigma$ for ethyl diazopropionate is similar to the better values in the compilation while that for 3 -diazo-butan-2-one is significantly lower. The standard errors, arising from the experimental scatter of the data, on

[^3]the fractionation factors and values of $\alpha$ from 13 differential experiments for diazo-ketone are also lower than those from 26 single experiments for the diazo-ester. Kresge ${ }^{2}$ and Gold ${ }^{12,15}$ have rightly emphasised the difficulty of determining fractionation factors from kinetic data. We believe that this work demonstrates the advantage of using the differential method to increase the precision of the kinetic data. For ethyl diazopropionate the difference in the fractionation factors caused by including the correction for the breakdown of the rule of the geometric mean or by changing the value of $l$ are comparable with the uncertainty from the scatter of the experimental data. However for 3-diazobutan-2-one the uncertainty is caused first by the correction for the rule of the geometric mean, secondly by the uncertainty in $l$, and only thirdly by the experimental scatter. The correction for the second transition state is rather unimportant and is less than the experimental scatter. The inclusion of the correction for the geometric mean gives a less good fit and a value of $\phi_{1}$ which is further from that found by product analysis; these differences however are not very significant. The rather reactant-like transition state may mean that the correction has been overestimated and there must still be some doubt as to the exact value of $p_{3}$.

We now turn to the values for $\phi_{1}$. A complication in the determination of $\phi_{1}$ from the products is that in these reactions two products are formed, ${ }^{9,16}$ as shown in Scheme 3. Our analysis has been confined to the


Scheme 3
major product, the hydroxy-compound. If there should be a large secondary isotope effect on the productdetermining step then the $\mathrm{D}: \mathrm{H}$ ratio in the intermediate would not be the same as that in either product. The correct expression is (22) where $\rho_{L}$ is given by (23).

$$
\begin{align*}
& \frac{\left[\mathrm{N}_{2}{ }^{+} \mathrm{CD}\right]}{\left[\mathrm{N}_{2}{ }^{+\mathrm{CH}]}\right]}=\frac{[\mathrm{LOCD}]}{[\mathrm{LOCH}]} \times \frac{\left(1+\rho_{\mathrm{D}}\right)}{\left(1+\rho_{\mathrm{H}}\right)}  \tag{22}\\
& \rho_{\mathrm{L}}=\frac{\left[\mathrm{CH}_{2} \mathrm{CL}\right]}{[\mathrm{LOCL}]} \tag{23}
\end{align*}
$$

We have measured $p_{L}$ to be $0.33 \pm 0.05$ for both H and D ; hence the correcting ratio is likely to be negligible and can at worse only be a few percent.

In Table 6 the values of $\phi_{1}$ for both compounds obtained from the independent sets of kinetic and product experiments are in good agreement. This

[^4]agreement as in previous work ${ }^{15,17,18}$ is consistent with the simple model of the $A-S_{\mathrm{E}} 2$ transition state, but it is possible that it is fortuitous; other models with or without extra fractionation factors could fit the data just as well. In view of the greater precision of our data for 3 -diazobutan-2-one it is worth exploring this point in more detail.
Instead of fitting the data to the simple model we write equation (24) where $n_{3}>n_{2}$. The left-hand side
$y_{x}=\left(1-x+\phi_{\mathrm{P}} x\right)(1-x+l x)^{3}[1-P x(1-x)] k_{x} / k_{0}$
\[

$$
\begin{equation*}
=\left(1-x+\phi_{2} x\right)^{n_{2}}\left(1-x+\phi_{3}\right)^{n_{2}} \tag{24}
\end{equation*}
$$

\]

contains the $\mathrm{L}_{3} \mathrm{O}^{+}$fractionation, the term for the proton transferred to carbon, and the geometric mean correction; $k_{x}$ has also been corrected for the second transition state. The remaining part of the isotope effect on the right-hand side is attributed to two as yet unspecified groups of factors. Following the procedure


Figure 3 Plots of $\gamma-1$ to find possible models for transition states with no correction for breakdown of the rate of the geometric mean. Horizontal solid lines represent $\gamma-1$ from data and equation (26) for different values of $n_{2}$. Parabolas represent $\gamma-1$ as a function of $\xi_{3}$ for different values of $\boldsymbol{n}_{2} / \boldsymbol{n}_{3}$; $\xi_{3}$ is defined by equation (28). Shaded rectangle is the area where possible transition states would have to lie if $\phi_{2}=l^{1-\alpha_{B}}$
suggested in a recent paper ${ }^{19}$ we fit $y$ to an equation of the form (25) and calculate the normalised curvature

$$
\begin{equation*}
y_{x}-1=a^{\prime \prime} x+b^{\prime \prime} x^{2} \tag{25}
\end{equation*}
$$

parameter (26). For $P=0, \gamma=0.51 \pm 0 \cdot 13$ and

$$
\begin{align*}
\gamma & =4 y_{2}\left(\mathbf{1}+y_{1}-2 y_{\frac{1}{}}\right) /\left(\mathbf{l}-y_{1}\right)^{2} \\
& =b^{\prime \prime}\left(2+a^{\prime \prime}+\frac{1}{2} b^{\prime \prime}\right) /\left(a^{\prime \prime}+b^{\prime \prime}\right)^{2} \tag{26}
\end{align*}
$$

for $P=0.03, \quad \gamma=0.75 \pm 0.13$. The parameter $\gamma$ describes how curved is the residual fractionation function containing $\phi_{2}$ and $\phi_{3}$. For a straight line $\gamma=0$, for a

[^5]perfect square $\gamma=0.5$, and for a ln curve $\gamma=1.0$. The results are shown in Figures 3 and 4 which show $\gamma$ as a function of $\xi_{3}$ (or $\xi_{2}$ ) where we have equations (27)
\[

$$
\begin{gather*}
\bar{\zeta}_{2}+\xi_{3}=1  \tag{27}\\
\xi_{m}=n_{m} \frac{\left(\phi_{m}-1\right)}{\left(\phi_{m}+1\right)} \frac{\left(2+a^{\prime \prime}+\frac{1}{2} b^{\prime \prime}\right)}{\left(a^{\prime \prime}+b^{\prime \prime}\right)} \\
 \tag{28}\\
\simeq n_{m}\left(1-\phi_{m}\right) /\left(1-y_{1}\right)
\end{gather*}
$$
\]

and (28). Figure 3 is for $P=0$, that is, no geometric mean correction, and Figure 4 for $P=0.03$.


Figure 4 Plots of $\gamma-1$ to find possible models for transition states with $P=0.03$ correction for breakdown of the rule of the geometric mean

Possible transition states are where the horizontal lines representing the values of $\gamma$ from the data and equation (26) intersect the parabolas; each parabola corresponds to a particular ratio $n_{2}: n_{3}$ (which is $<1$ ). In particular the parabola for which $n_{3} \longrightarrow \infty\left(n_{2} / n_{3}=\right.$ 0 ) represents a contribution from a generalised medium effect on either the reactants or the transition state. ${ }^{19}$ The scale on the $y$ axis is in units of $n_{2}{ }^{-1}$ so there is a horizontal line for $\gamma$ for each value of $n_{2}$; hence the intersection of each line and each parabola defines the values of $n_{2}$ and $n_{3}$. The fractionation factors are given from the value of $\xi_{3}$ by equations (27) and (28). It is likely that the factors are less than unity so the area of possible solutions is bounded by $\xi_{3}=0$ (or $\xi_{2}=1$ ) and $\xi_{3}=1$ (or $\xi_{2}=0$ ).
The error on $\gamma$ is rather large, so our conclusions must be correspondingly tentative. For both values of $P$ it appears that $n_{1} \gtrless 4$; the point $\xi_{3}=1, \gamma-1=0$, which would ascribe the whole of the effect to a generalised medium effort, is an improbable solution. The characteristics of different transition states are given

[^6]in Table 7. C in Figure 3 is interesting since it is a possible alternative model to the simple one shown in

Table 7
Possible transition states from Figures 3 and 4
Figure 3

|  | $n_{2}$ | $\phi_{2}$ | $n_{3}$ | $\phi_{s}$ | $\phi_{3}{ }^{\text {n }}$ | Comments |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| A | 2 | 0.78 | ? |  | 1.00 | Simple transition state |
| B | 1 | $0 \cdot 69$ | 4 to $\infty$ |  | 0.88 | Single factor of 0.69 unlikely |
| C | 1 | 0.72 | 2 | 0.92 |  | Improbable, see text. |
| D | 1 | 0.74 | 1 | $0 \cdot 82$ |  | Two unequal single factors are improbable. |
| E | 1 | 0.78 | 1 | 0.78 |  | Simple transition state. |
| F | 1 | 1.00 | 2 | 0.78 |  | Simple transition state. |

Figure 4

| A | 4 | 0.88 | ? |  | 1.00 | 4 such factors are improbable. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| B | 2 | 0.83 | 4 to $\infty$ |  | 0.88 | Simple transition state + |
| C | 2 | $0 \cdot 86$ | 2 | 0.90 |  | Second pair unlikely. |
| D | 2 | 0.88 | 2 | 0.88 |  | See A |
| E | 2 | 1.00 | 4 | 0.88 |  | See A |
| F | 1 | 0.76 | $\infty$ |  | 0.78 | Single factor unlikely and too large a medium effect |
| G | 1 | 0.84 | 4 | 0.92 |  | All factors somewhat improbable. |

structure (I). However, a value of $\phi_{2}$ as large as 0.72

suggests that this proton is in fact bound to the second oxygen, in which case this model reverts to the simple one. The two non-transferring protons on the second oxygen could only have different factors if the energy redistribution was slow compared with the second proton transfer. Such effects are possible in low-pressure gas reactions but are unlikely in solution and in particular in this case if the $\phi_{1}$ proton has time to receive energy from the $\phi_{2}$ proton then so does the third proton.

The argument for the simple model for the $A-S_{\mathrm{E}} 2$ transition state has hitherto rested uncomfortably on Ockham's razor. This analysis does suggest that the most probable models are indeed $\mathrm{A}, \mathrm{E}$, or F , the simple transition state, in Figure 3 or $B$ in Figure 4. The solvation factors for the transition state in $B, \Pi \phi_{3}=$ $0 \cdot 88$, are about as large as one would expect remembering that the solubility of 3-diazobutan-2-one does not vary greatly between $\mathrm{H}_{2} \mathrm{O}$ and $\mathrm{D}_{2} \mathrm{O}$ and that the reaction only involves cations and neutral species. ${ }^{12}$ These solvation factors are nearly unity if, instead of taking the most probable value of $\phi_{\mathbf{P}}=0 \cdot 208$, one takes a value one standard deviation away from the mean $\phi_{\mathrm{P}}=0.216$.

20 E. A. Halevi, Solvent Isotope Effects, 22nd Farkas Memorial Symposium, 1970, to be published.

In Table 8 we have collected values of $\alpha_{L+}$ to compare them with $\alpha_{B}$, the Brønsted $\alpha$ for carboxylic acids. We also include Kreevoy and Konasevich's similar data for diazoacetate anion. ${ }^{20-22}$ The first thing to note is how the inclusion of $\Pi \phi_{3}=0.88$ has a drastic effect on $\alpha_{L^{+}}$. Depending on their temperaments previous workers have either despaired ${ }^{20}$ or have assumed

## Table 8

Values of $\alpha$, from this work if no reference given

|  | $P$ | $\Pi \phi_{\mathbf{z}}$ | $\begin{gathered} \alpha_{L^{+}} \\ \text {from } \\ \text { equation } \\ (20) \end{gathered}$ | Ref. | $\begin{gathered} \alpha_{\mathbf{B}} \\ \text { for } \\ \mathrm{RCO}_{\mathbf{2}} \mathrm{H} \end{gathered}$ | Ref. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{N}_{2} \mathrm{CHCO}_{2}-$ |  | 1.00 | $0 \cdot 30 \pm 0 \cdot 10^{*}$ | 19 | $0.51 \pm 0.03$ | 22 |
| 3-Diazobutan-2one | 0.00 | 1.00 | $0.31 \pm 0.01{ }_{4}$ |  |  |  |
|  | 0.03 | 1.00 | $0.27 \pm 0.01$ 。 |  | $0.61 \pm 0.03$ | 9 |
|  | 0.03 | 0.88 | $0.50 \pm 0.02$ |  |  |  |
| Ethyl diazopropionate | 0.00 | 1.00 | $0.35 \pm 0.04$ |  | $0.59 \pm 0.04$ | 9 |
|  | 0.03 | 1.00 | 0.30士 0.04 |  |  |  |

* Standard error estimated from errors quoted, $\mathbf{4 \cdot 2} \pm \mathbf{0 . 1 0}$ being taken for product analysis.
optimistically that $\Pi \phi_{3}=1 \cdot 00 .^{15,17,18,21}$ We have tried in our analysis to show how one might obtain some information about $\Pi \phi_{3}$. We cannot yet claim to have been completely successful. To match the improved precision of the kinetic data we now need more precise values for $\phi_{\mathrm{P}}, l$, and, most important, direct experimental confirmation of the theoretical value of $p_{3}$.

Table 8 shows that the values of $\alpha_{L^{+}}$are less than the values for $\alpha_{\mathrm{B}}$. We have shown this for 3-diazobutan2 -one in another way by plotting in Figure 3, the shaded rectangles in which the transition states would have to lie if $n_{2}=2$ and equation (29) holds for a range

$$
\begin{equation*}
\phi_{2}=l^{1-(0.61 \pm 0.03)} \tag{29}
\end{equation*}
$$

of $\gamma$ within one standard deviation of the mean. No models can be found in Figure 3. In Figure 4 models with $n_{3}>4$ represent a solvation contribution of $\Pi \phi_{i}^{\ddagger}=0.77$; this seems to us implausibly large. We did consider transition state C and whether it might be structure (II), but this is unlikely in view of the analysis

in Part $\mathrm{II},{ }^{13}$ in which the carbon-nitrogen bond is broken in a second separate transition state.

Thus we believe that we have shown that $\alpha_{L^{+}}$is significantly less than $\alpha_{B}$ even when one allows a reasonable amount of solvation fractionation in the transition

[^7]state. This is experimental confirmation of an earlier warning ${ }^{11}$ of the dangers of calculating $\phi_{2}$ from equation (20) and $\alpha_{B}$. This point is further discussed in Part V, and the reasons for the differences in $\alpha$ are discussed in Part VI.

## APPENDIX 1

This contains the equations for working out the values and standard errors of $\theta_{1}$ and $\theta_{2}$ from a set of $n$ values of $y$ and $x$ in equations (6) and (7). In equation (7), $a$ and $b$ are given by (30) and (31) where $X=\left(\Sigma x^{3}\right)^{2}-\Sigma x^{4} \Sigma x^{2}$;

$$
\begin{align*}
& a=X^{-1}\left(\Sigma x^{3} \Sigma x^{2} y-\Sigma x^{4} \Sigma x y\right)  \tag{30}\\
& b=X^{-1}\left(\Sigma x^{3} \Sigma x y-\Sigma x^{2} \Sigma x^{2} y\right) \tag{31}
\end{align*}
$$

$\theta_{1}$ and $\theta_{2}$ can then be worked out from equation (8).
To work out the errors we need the functions (32) - (36) from which we obtain equation (37), components of which are given by equations (38)-42).

$$
\begin{align*}
\Sigma d_{i}^{2}= & a^{2} \Sigma x^{2}+2 a b \Sigma x^{3}+b^{2} \Sigma x^{4}-2 a \Sigma x y- \\
& 2 b \Sigma x^{2} y+\Sigma y^{2}  \tag{32}\\
E_{y}{ }^{2} & =\Sigma d^{2} /(n-2)  \tag{33}\\
E_{a}^{2} & =E_{y}{ }^{2}\left(\Sigma x^{4} / X\right)  \tag{34}\\
E_{b}{ }^{2} & =E_{y}{ }^{2}\left(\Sigma x^{2} / X\right)  \tag{35}\\
E_{a b} & =E_{y}{ }^{2}\left(\Sigma x^{3} / X\right)  \tag{36}\\
E_{\theta_{n}}{ }^{2} & =Z_{a . n^{2} E_{a}^{2}+2 Z_{a . n} Z_{b . n} E_{a b}+2_{b . n}{ }^{2} E_{b}{ }^{2}}^{Z_{a .1}}=\frac{1}{3}(1+2 a / r)  \tag{37}\\
Z_{a .2} & =\frac{1}{3}(1-a / r)  \tag{38}\\
Z_{b .1} & =1 / r  \tag{39}\\
Z_{b, 2} & =-1 / 2 r  \tag{40}\\
r & =\sqrt{ }\left(a^{2}+3 b\right) \tag{42}
\end{align*}
$$

## APPENDIX 2

This contains the equations for working out the values and standard errors of $k_{0}, \theta_{1}$, and $\theta_{2}$ from a set of $n$ values
of $y$ and $x$ in equations (11) and (12). In equation (12) we have the relationships (43)-(46). In equation (8)

$$
\begin{array}{r}
c^{\prime}=X^{-1}\left|\begin{array}{ccc}
\Sigma y & \Sigma x y & \Sigma x^{2} y \\
\sum x & \Sigma x^{2} & \Sigma x^{3} \\
\Sigma x^{2} & \Sigma x^{3} & \Sigma x^{4}
\end{array}\right| \\
a^{\prime}=X^{-1}\left|\begin{array}{ccc}
\Sigma y & \Sigma x y & \Sigma x^{2} y \\
\Sigma x^{2} & \Sigma x^{3} & \Sigma x^{4} \\
n & \Sigma x & \Sigma x^{2}
\end{array}\right| \\
b^{\prime}=X^{-1}\left|\begin{array}{ccc}
\Sigma y & \Sigma x y & \Sigma x^{2} y \\
n & \Sigma x & \Sigma x^{2} \\
\Sigma x & \Sigma x^{2} & \Sigma x^{3}
\end{array}\right| \\
\quad X=\left|\begin{array}{ccc}
n & \Sigma x & \Sigma x^{2} \\
\Sigma x & \Sigma x^{2} & \Sigma x^{3} \\
\Sigma x^{2} & \Sigma x^{3} & \Sigma x^{4}
\end{array}\right| \tag{46}
\end{array}
$$

$a=-b^{\prime} / a^{\prime}$ and $b=-c^{\prime} / a^{\prime}$; hence values of $\theta_{1}$ and $\theta_{\mathbf{2}}$ can be found.
For the errors we need the functions (47) and (48)-(54), from which we obtain equation (55), where $Z_{a, n}$ and $Z_{b, n}$ have the same values as in Appendix 1, and relations (56) and (57) apply.

$$
\begin{align*}
& \Sigma d^{2}=n\left(c^{\prime}\right)^{2}+2 b^{\prime} c^{\prime} \Sigma x+\left[\left(a^{\prime}\right)^{2}+2 b^{\prime} c^{\prime}\right] \Sigma x^{2}+2 a^{\prime} b^{\prime} \Sigma x^{3} \\
& +\left(b^{\prime}\right)^{2} \Sigma x^{4}-2 c^{\prime} \Sigma y-2 a^{\prime} \Sigma x y-2 b^{\prime} \Sigma x^{2} y+\Sigma y^{2} \quad \text { (47) } \\
& E_{y^{2}}=\Sigma d^{2} /(n-3)  \tag{48}\\
& E_{c^{\prime}}{ }^{2}=E_{y}{ }^{2} X^{-1}\left[\Sigma x^{2} \Sigma x^{4}-\left(\Sigma x^{3}\right)^{2}\right]  \tag{49}\\
& E_{a^{2}}{ }^{2}=E_{\nu}{ }^{2} X^{-1}\left[n \Sigma x^{4}-\left(\Sigma x^{2}\right)^{2}\right]  \tag{50}\\
& E_{b^{\prime}}{ }^{2}=E_{y}{ }^{2} X^{-1}\left[n \Sigma x^{2}-(\Sigma x)^{2}\right]  \tag{51}\\
& E_{c^{\prime} a^{\prime}}=E_{y^{2}}{ }^{2} X^{-1}\left[\Sigma x^{3} \Sigma x^{2}-\Sigma x \Sigma x^{4}\right]  \tag{52}\\
& E_{a^{\prime} b^{\prime}}=E_{y}{ }^{2} X^{-1}\left[\Sigma x^{2} \Sigma x-n \Sigma x^{3}\right]  \tag{53}\\
& E_{c^{\prime} b^{\prime}}=E_{y}{ }^{2} X^{-1}\left[\Sigma x \Sigma x^{3}-\left(\Sigma x^{2}\right)^{2}\right]  \tag{54}\\
& E_{\theta_{n}}{ }^{2}=c^{\prime-2}\left[Z_{a . n}{ }^{2} E_{a^{\prime}}{ }^{2}+Z_{b . n}{ }^{2} E_{b^{\prime}}{ }^{2}+Z_{c . n}{ }^{2} E_{c^{\prime}}+\right. \\
& \left.2 Z_{a . n} Z_{b . n} E_{a^{\prime} b^{\prime}}+2 Z_{a . n} Z_{c . n} E_{c^{\prime} a^{\prime}}+2 Z_{b . n} Z_{c . n} E_{c^{\prime} b^{\prime}}\right]  \tag{55}\\
& Z_{c .1}=\theta_{1}-b / r  \tag{56}\\
& Z_{c .2}=\theta_{2}+\frac{1}{2} b / r \tag{57}
\end{align*}
$$

[2/448 Received, 28th February, 1972]


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